Last Time: Configuration Space $\longrightarrow$ Choset, et al. chpt 2/3

Q denote configuration space

Examples

$Q = \mathbb{R}$: translation in plane

$Q =$ Torus : 2-link robot arm

$Q = SE(2) = SO(2) \times \mathbb{R}^2 \Rightarrow$ $\boxed{\text{rotate + translate in the plane}}$

can approximate fairly well by $Q = \mathbb{R}^2$ using appropriate "bounding volume" for the robot.

$SE(3)$: Drone, submarine

$T^n \Rightarrow$ robot arms w/revolute joints

Mobile manipulators: $SE(2) \times T^N$

Paths

$\gamma : [0,1] \to Q$, γ smooth, continuous, or differentiable

$\gamma(t)$ denotes configur along path at time t.

s.t. $\gamma(0) = q_{init}$

$\gamma(1) = q_{goal}$ (2 pt boundary value problem)

collision-free: $q \in \underline{Q_{free}}$ for $t \in [0,1]$

↳ more soon ....

$\|\dot{\gamma}\| \leq V_{max}$

$\|\ddot{\gamma}\| \leq Acc_{max}$

Path Planning: Find γ s.t $\gamma \in Q_{free}$, $\gamma(0) = q_{init}$, $\gamma(1) = q_{goal}$

Q_{free} = free configuration space

Let $\underline{A(q)}$ denote points occupied by a robot, A, at configuration q.

$$Q_{obst} = \{ q \in Q \mid A(q) \cap \mathcal{O} \neq \emptyset \},$$

$\mathcal{O}$ denotes obstacles in workspace.

$$Q_{free} = Q \setminus Q_{obst}$$

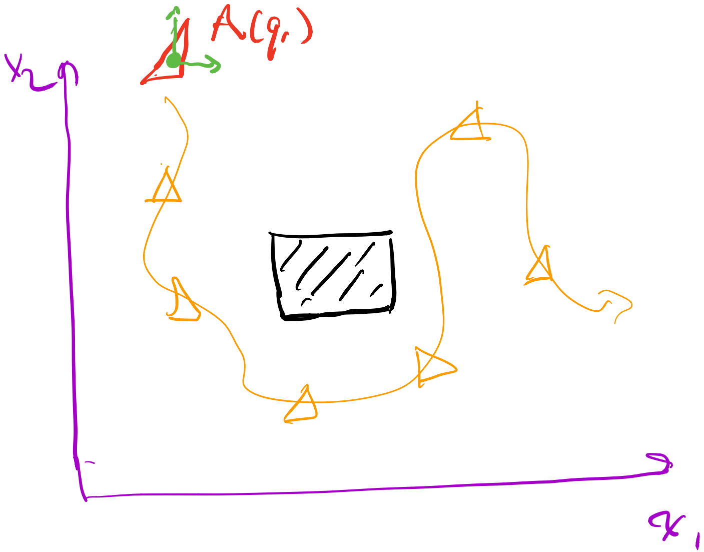
$$\overline{Q}_{free} = \text{semi-free config. space} = Q_{free} \cup \underbrace{\partial}_{\text{boundary}}(Q_{free})$$

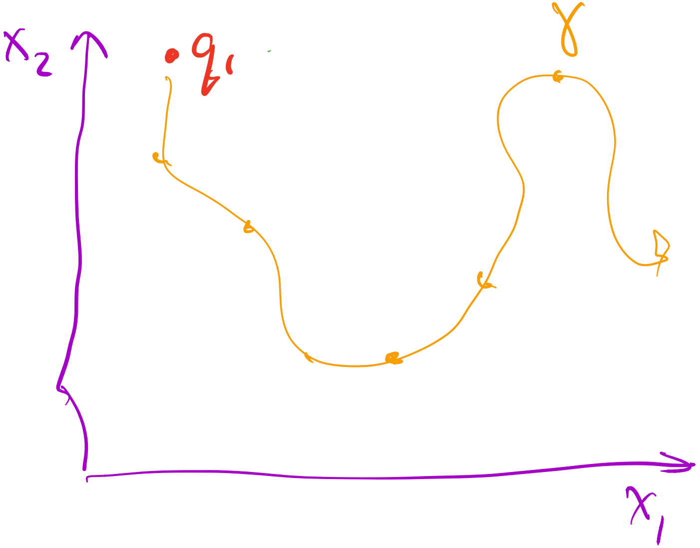
Example

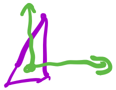
→ Robot

Workspace $= \mathbb{R}^2$

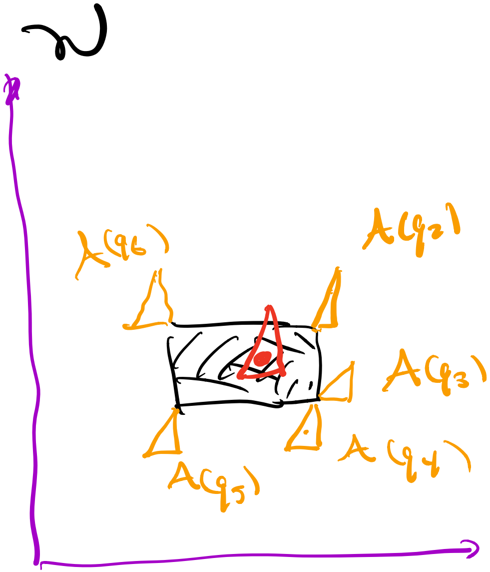
$Q = \mathbb{R}^2$

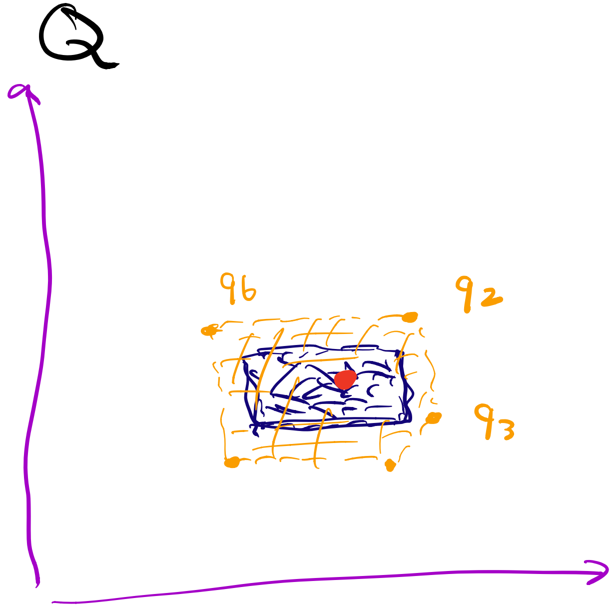






$\theta = 0$



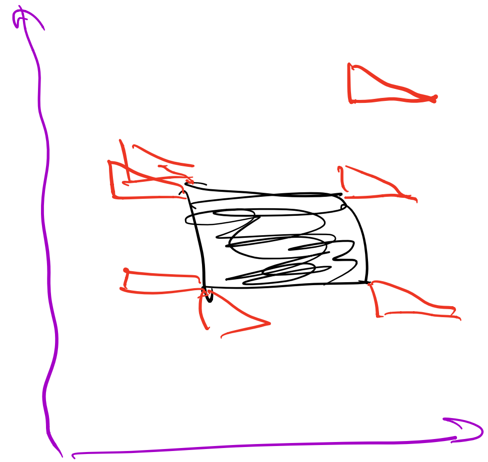


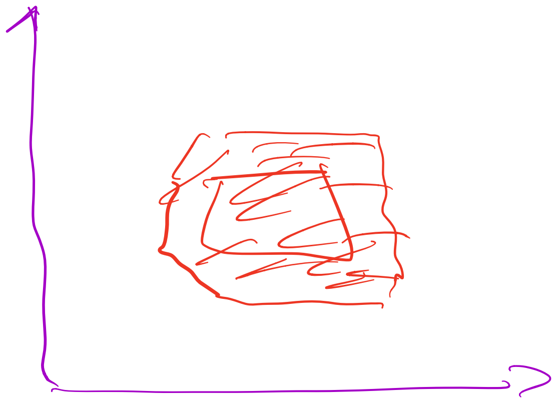
q s.t. $q = (x_1, x_2) \in \mathcal{O}$

$\boxed{\text{Stack these for } \theta = 1, 2, \ldots\ 360}$

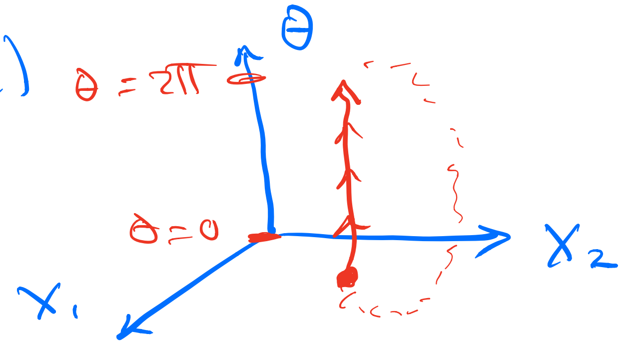
approximates $Q = SE(2)$

$\theta = 90$





$Q = SE(2)$



Now... How to plan paths

Two Methods for $Q = \mathbb{R}^2$

- Visibility Graph
- Cell decomposition

$\Longrightarrow$ Both Build a Graph $G = (V, E)$

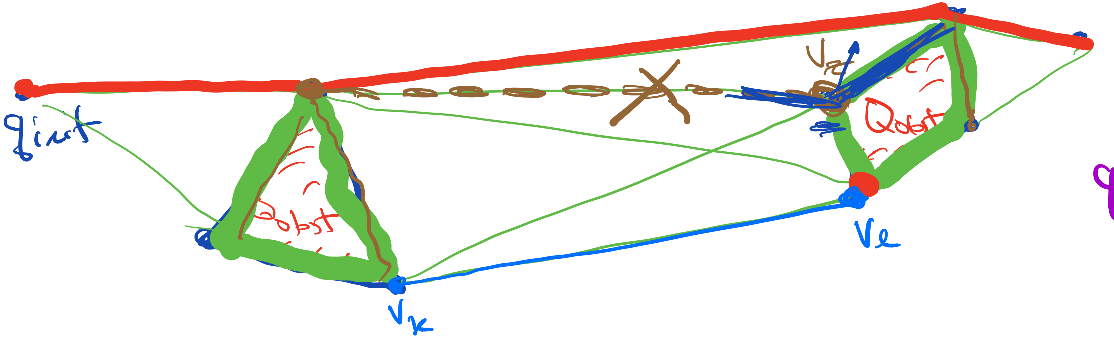
V = set of configurations

E = set of straight-line paths,
free, semi-free path

Visibility graph

V = set of obsta vertices + q_{init} + q_{goal}

$E \Rightarrow \langle v_i, v_j \rangle$ s.t. $\overline{v_i v_j} \subseteq \overline{Q}_{free}$



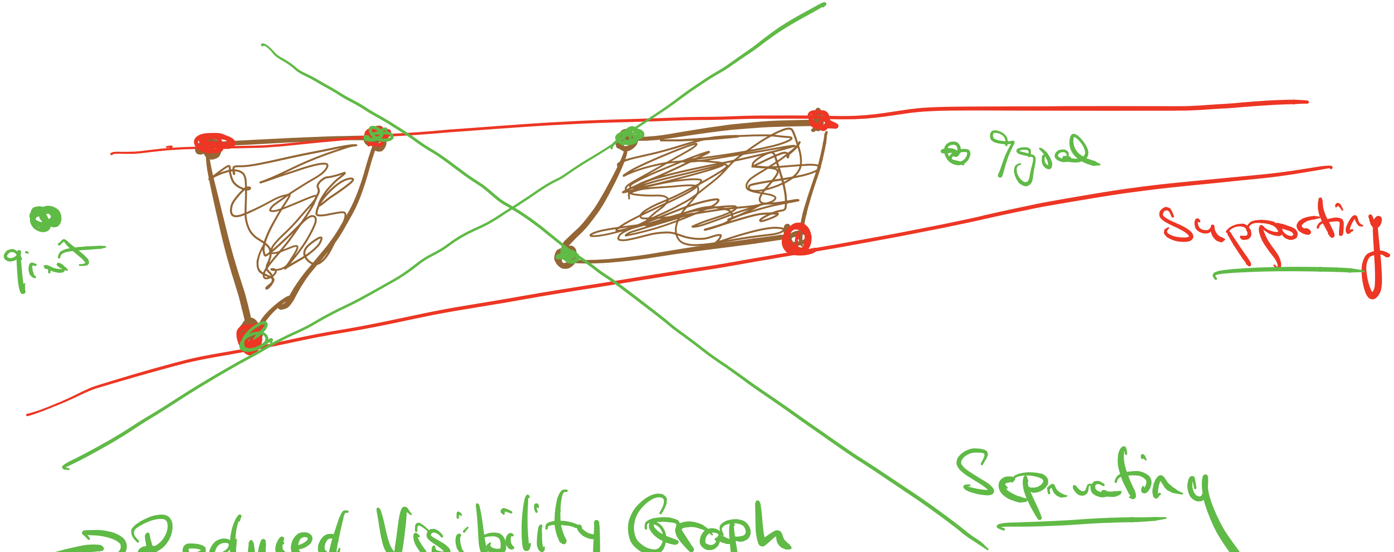
Shortest path from q_{init} to $q_{goal} \in VG$

Only need $V_k V_l$ that are tangent to obstacles

Edge E_i
Vertices $V_k V_l$

Does segment $\overline{V_k V_l}$ intersect edge E_i

For two convex polygons, only four edges



$\Longrightarrow$ Reduced Visibility Graph